



APPLICATIONS OF MATRIX MATHEMATICS

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ABSTRACT

Applied Mathematics is future classified as vector algebra, differential calculus, integration, discrete Mathematics, Matrices & determinant etc. Among various topic matrices is generally interesting. Matrices have a long history of application in solving linear equations. Matrices are incredibly useful things that crop up in many different applied areas.

Matrix mathematics applies to several branches of science, as well as different mathematical disciplines. Engineering Mathematics is applied in our daily life. We see the results of matrix in every computer-generated image that has a reflection or distortion effects such as light passing through rippling water. Before computer graphics, the science of optics used matrix to account for reflection and for refraction. In mathematics, one application of matrix notation supports graph theory. In an adjacency matrix, the integer value of each element indicates how many connections a particular node has.

KEYWORDS: Matrices, calculus, determinant, reflection, refraction, cryptography.

INTRODUCTION:

A Matrices is a two dimensional arrangement of numbers in row and column enclosed by a pair of square brackets or can say matrices are nothing but the rectangular arrangement of numbers, expression, symbols which are arranged in columns and rows. Most commonly, a matrix over a field F is a rectangular array of scalars each of which is a member of F. The individual items in a matrix are called its elements or entries. Provided that they are the same size i.e. the same number of rows and the same number of columns, two matrices can be added or subtracted element by element. The rule for matrix multiplication, however, is that two matrices can be multiplied only when the number of columns in the first equals the number of rows in the second. Any matrix can be multiplied element-wise by a scalar from its associated field. A major application of matrices is to represent linear transformations, that is, generalizations of linear functions such as $f(x) = 2x$. For example, the rotation of vectors in three dimensional space is a linear transformation which can be represented by a rotation matrix R, if v is a column vector (a matrix with only one column) describing the position of a point in space, the product Rv is a column vector describing the position of that point after a rotation. The product of two transformation matrices is a matrix that represents the composition of two linear transformations. Another application of matrices is in the solution of systems of linear equations. If the matrix is square, it is possible to deduce some of its properties by computing its determinant. For example, a square matrix has an inverse if and only if its determinant is not zero. Insight into the geometry of a linear transformation is obtainable from the matrix's Eigen values and Eigen vectors.

A major branch of numerical analysis is devoted to the development of efficient algorithms for matrix computations, a subject that is centuries old and is today an expanding area of research. Matrix decomposition methods simplify computations, both theoretically and practically. Algorithms that are tailored to particular matrix structures, such as sparse matrices and near-diagonal matrices, expedite computations in finite element method and other computations. Infinite matrices occur in planetary theory and in atomic theory. A simple example of an infinite matrix is the matrix, representing the derivative operator, which acts on the Taylor series of a function.

Matrices find many applications in scientific fields and apply to practical real life problems as well, thus making an indispensable concept for solving many practical problems. Matrices are used in representing the real world data's like the traits of people's population, habits, etc. Matrices can be solved physical related application and one applied in the study of electrical circuits, quantum mechanics and optics, with the help of matrices, calculation of battery power outputs, resistor conversion of electrical energy into another useful energy, these matrices play a role in calculation. Matrix mathematics simplifies linear algebra, at least in providing a more compact way to deal with groups of equations in linear algebra. Some properties of matrix mathematics are important in math theory.

HISTORY:

Matrices have a long history of applications in solving linear equations, between 300BC and AD200. The first example of the use of matrix methods to solve simultaneous equations including the concept of determinants, easily matrix theory emphasizes determinants more strongly than matrices and an independent matrix concept akin to the modern notion emerged only in 1858. With Cayley's

Memor is on the theory of matrices, the term matrix was coined by Sylvester, Who understood a matrix as an object giving rise to number of determinants today called minors. First concept of Mathematics was applied on around 1850AD but its uses were applicable in ancient era. The Latin word of matrix means womb. It can also mean more generally any place which something form or produced.

Application of Matrices:

- In the field of computing, matrices are used in message encryption. They are used to create three-dimensional graphic images and realistic looking motion on a two-dimensional computer screen and also in the calculation of algorithms that create Google page rankings.
- Matrices are used to compress electronic information and play a role in storing fingerprint information.
- In solving the problems using Kirchhoff's Laws of voltage and current, the matrices are essential.
- Errors in electronic transmissions are identified and corrected with the use of matrices.
- In the calculation of battery power outputs, resistor conversion of electrical energy into another useful energy, these matrices play a major role in calculations.
- The matrix calculus is used in the generalization of analytical notions like exponentials and derivatives to their higher dimensions.
- Matrices and their inverse matrices are used for a programmer for coding or encrypting a message.
- A message is made as a sequence of numbers in a binary format for communication and it follows code theory for solving.
- With these encryptions only, internet functions are working and even banks could work with transmission of sensitive and private data's.
- In geology, matrices are used for taking seismic surveys.
- Matrices are used for plotting graphs, statistics and also to do scientific studies in almost different fields.
- Matrices are best representation methods for plotting the common survey things.
- Matrices are used in calculating the gross domestic products in economics which eventually helps in calculating the goods production efficiently.
- Matrices are used in many organizations such as for scientists for recording the data for their experiments.

- In robotics and automation, matrices are the base elements for the robot movements.
- The movements of the robots are programmed with the calculation of matrices rows and columns.
- The inputs for controlling robots are given based on the calculations from matrices.
- In the field of medicine, CAT scans and MRI's use matrices.
- In physics, matrices are used to study electrical circuits and quantum mechanics and optics.
- Matrix arithmetic helps us calculate the electrical properties of a circuit with voltage, amperage, resistance, etc.
- The science of optics used matrix mathematics to account for reflection and for refraction.
- Computers run Markov simulations based on stochastic matrices in order to model events ranging from gambling through weather forecasting to quantum mechanics.
- Matrices are used to represent real-world data about specific populations, such as the number of people who have a specific trait. They can also be used to model projections in population growth.
- Matrices are used in calculating the gross domestic products in economics which eventually helps in calculating the goods production efficiently.
- Matrices are used in many organizations such as for scientists for recording the data for their experiments.
- Matrices are used to cover channels, hidden text within web pages, hidden files in plain sight, null ciphers and steganography.
- In recent wireless internet connection through mobile phone, known as wireless application protocol also utilize matrices in the form of stenography.
- Cryptography also utilize matrices, cryptography is science of information security. These technologies hide information in storage or transits.
- They are best representation methods for plotting the common survey things.
- Stochastic matrices and Eigen vector solvers are used in the page rank algorithms which are used in the ranking of web pages in Google search.

Matrices-Application to Cryptography

The basic idea of cryptography is that information can be encoded using an encryption scheme and decoded by anyone who knows the scheme. There are lots of encryption schemes ranging from very simple to very complex. Most of them are mathematical in nature.

Today, sensitive information is sent over the Internet every second, credit card numbers, personal information, bank account numbers, letters of credit, passwords for important databases, etc. Often, that information is encoded or encrypted.

The encoder is a matrix and the decoder is its inverse. Let A be the encoding matrix, M the message matrix, and X will be the encrypted matrix (the sizes of A and M will have to be consistent and will determine the size of X). Then, mathematically, the operation is

$$AM = X$$

Someone has X and knows A, and wants to recover M, the original message. That would be the same as solving the matrix equation for M. Multiplying both sides of the equation on the left by A^{-1} we have

$$M = A^{-1}X$$

(Note: A must have an inverse)

Example: Let A=1, B=2, C=3, and so on, let a blank be represented by 0.

Let's encode the message "THE EAGLE HAS LANDED". We need to translate letters into numbers. Using the list above, the message becomes:

20, 8, 5, 0, 5, 1, 7, 12, 5, 0, 8, 1, 19, 0, 12, 1, 14, 4, 5, 4

Now we need to decide on a coding matrix.

$$A = \begin{bmatrix} 3 & 0 & 1 & 1 \\ 1 & 2 & 5 & 0 \\ 1 & 1 & 3 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}$$

Since this is a 4 x 4 matrix, we can encode only 4 numbers at a time. We break the message into packets of 4 numbers each, adding blanks to the end if necessary. The first group is 20, 8, 5, and 0. The message matrix will be 4 x 1.

$$\begin{bmatrix} 3 & 0 & 1 & 1 \\ 1 & 2 & 5 & 0 \\ 1 & 1 & 3 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix} \left| \begin{array}{c} 20 \\ 8 \\ 5 \\ 0 \end{array} \right. = \begin{bmatrix} 65 \\ 61 \\ 43 \\ 45 \end{bmatrix}$$

So the first 4 encrypted numbers are 65, 61, 43, and 45

Next 4 encrypted numbers are 5, 1, 7, and 12

$$\begin{bmatrix} 3 & 0 & 1 & 1 \\ 1 & 2 & 5 & 0 \\ 1 & 1 & 3 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix} \left| \begin{array}{c} 5 \\ 1 \\ 7 \\ 12 \end{array} \right. = \begin{bmatrix} 34 \\ 42 \\ 27 \\ 29 \end{bmatrix}$$

The second group is 34, 42, 27, and 29. Notice that 5 came out as 43 in the first group, but as 34 in the second group. That's one of the advantages of the matrix scheme. The same data can be encoded different ways making it harder to find a pattern.

Encoding the entire sequence gives us the encrypted message:

65, 61, 43, 45, 34, 42, 27, 29, 24, 45, 29, 19, 70, 79, 55, 51, 51, 47, 33, 37

Let's decode it using the inverse matrix

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 2 & 3 & -5 & -2 \\ -1 & -1 & 2 & 1 \\ -1 & 1 & -2 & 2 \end{bmatrix}$$

Decoding the first 4 numbers, we have

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 2 & 3 & -5 & -2 \\ -1 & -1 & 2 & 1 \\ -1 & 1 & -2 & 2 \end{bmatrix} \left| \begin{array}{c} 65 \\ 61 \\ 43 \\ 45 \end{array} \right. = \begin{bmatrix} 20 \\ 8 \\ 5 \\ 0 \end{bmatrix}$$

The first 4 numbers decode as the first 4 numbers in the original message.

Matrix encryption is just one of many schemes. Every year, the National Security Agency, the military and private corporations hire hundreds of people to devise new schemes and decode existing ones.

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